ON THE HOMOGENEITY OF TYPE IA SUPERNOVAE LIGHT CURVES *

FREDERICK J. MAYER
Mayer Applied Research Inc., 1417 Dicken Drive, Ann Arbor, MI 48103

JOHN R. REITZ
2260 Chaucer Court, Ann Arbor, MI 48103

(Received 13 June 2001;; accepted 2 July 2002)

Abstract. A self-similar, hydrodynamic model is derived and used to generate SNe light-curves. It is found that the temporal development of the SN light-curve is governed by a 'dynamic time' parameter, and that the observed near-identical, normalized light-curves of Type Ia SNe suggest that they have evolved from progenitor stars of the same central density. Fitting the model parameters to observed Type Ia SNe light-curves suggests that the SNe have originated from the same mass progenitors. The model also provides a theoretical basis for the Phillips observation relating the absolute magnitude of the Type Ia SN to its half-width.

Keywords: type Ia supernovae, light curves

1. Introduction

Observations over the past few years have shown that many Type Ia supernovae have such similar 'light-curve' shapes that, with amplitude and time re-scaling algorithms (see G. Goldhaber et al., 2001; SCP, 1998 and A. Reiss et al., 1998), all of the normalized light-curve data can be plotted on one curve. Such a remarkable effect should find its explanation in the basic explosive hydrodynamics of the supernova process and perhaps a single parameter is responsible for the data 'homogeneity'. In this paper, we suggest a mechanism by which this homogeneity occurs. To understand the mechanism, we extend our earlier similarity model (Mayer and Reitz, 2001, herein MRa) and present a simplified model [herein MRb] for calculating Type Ia SNe light-curves. In MRb, we find a characteristic time, the 'dynamic time', such that the calculated light-curves of SNe originating from progenitor stars with the same central density produce the light-curve shapes that agree well with the experimental data.

We should point out at the outset that our model does not examine the details of the thermonuclear explosion or core collapse which initiates the supernova. Many papers have addressed this phase of the problem (see e.g., Arnett, 1996, and references therein). Our problem starts with a progenitor star which receives a specified

* Thanks to Mario Livio for helpful suggestions.

energy per unit mass (E_1/M_0), presumably from a thermonuclear explosion, in a short time (on the order of 100 seconds) and expands against gravity. Our goal is to determine to what extent the conservation of mass, energy and momentum limit the domain of SNe parameters as evidenced through observation of their light-curves.

Although the model is a simple one, it is the simplicity that provides the connection between the various phenomena occuring in the SN explosion/expansion over the enormous dynamic range of time and distance scales.

In section 2, we describe the MRb similarity model. In section 3, we examine the characteristic time parameter and its influence on the light-curves generated using the model. Finally, we discuss some implications in section 4.

2. Derivation of the Simplified Similarity Model – MRb

We previously derived a similarity model (MRa) for the explosive hydrodynamics of a supernova. The model conserved mass, momentum, and energy, and modeled the large-scale dynamics with a Gaussian density profile $\rho(r,t)$, a linear velocity profile $v(r,t)$, and a time-dependent scale-height $y(t)$. After all shocks have 'rung-out' and the detonation energy is spread over the debris mass, the hydrodynamics tends toward (in a spherically symmetric explosion into a vacuum) a Gaussian shape (see Sedov, also Mayer and Tanner, referenced in MRa).

$$\rho(r,t) = \rho_0\,[\exp -(r/r_s)^2]\,y^{-3} \qquad v(r,t) = (r/r_s)\,\dot{r}_s \tag{1}$$

where

$$r_s = r_s(t) = r_0\,y(t). \tag{2}$$

Here, r_0 is the starting radial mass scale of the progenitor star. We noted in MRa that $r_0 = 0.236 R_0$ where R_0 is the progenitor star's initial radius and the progenitor's mass is $M_0 = \pi^{3/2}\rho_0 r_0^3$.

After doing numerous numerical integrations of our MRa model, it was clear that a considerable simplification could be introduced by eliminating the gas pressure compared to the radiation pressure. It is easy to show that, throughout a typical SN explosion, the radiation pressure is about 100 times (or more) larger than the particle pressure in all realistic cases. The radiation pressure is given by,

$$p = (1/3)\,a\,\theta^4(t)\exp[-(r/r_s)^2]$$

where $\theta(t)$ is the central temperature of the star. So, using our previous notation, we start with the radial momentum equation:

$$\rho\,\frac{dv}{dt} = -\frac{\partial p}{\partial r} + F_g \tag{3}$$

where p is the pressure and F_g is the gravitational force per unit volume. To this we add the energy consevation equation. This gives us Eqs. (11) and (12) of MRa (without the particle pressure terms) listed again as Eq.(4) and (5) below:

$$\frac{1}{3}\frac{a\,\theta^4\,y^3}{\rho_0} = \frac{r_0^2}{2}\left[y\,\ddot{y} + 2\sqrt{2}/(\tau^2\,y)\right] \tag{4}$$

and

$$\frac{d}{dt}\left(\frac{a\,\theta^4\,y^3}{\rho_0}\right) - \frac{p}{\rho^2}\frac{\partial\rho}{\partial t} - \frac{d}{dt}\left(\frac{GM_0}{\sqrt{2\pi}\,r_0\,y}\right) = \frac{\dot{E}(t)}{M_0} \tag{5}$$

τ is defined below, Eq.(9), and the internal energy release function is given by $E(t) = E_1 f(t)$ where $f(t) = 1 - e^{-t/t_1} + f_{\rm rad}$ with

$$f_{\rm rad} = \delta\left[1 - e^{-t/t_2} + 2.2\left(1 + 0.085\,e^{-t/t_2} - 1.085\,e^{-t/t_3}\right)\right] \tag{6}$$

the relatively slow radioactive heating.

Eliminating $\theta(t)$ as a variable between the equations, we obtain the following uncoupled ordinary differential equation for the scale-height $y(t)$,

$$y\ddot{y} + \frac{\dot{y}^2}{2} = \frac{2\,E_1}{3M_0r_0^2}f(t) - \frac{\sqrt{2}GM_0}{3\sqrt{\pi}r_0^3}(1 - \frac{1}{y}) \tag{7}$$

with the boundary conditions $y(0) = 1$, $\dot{y}(0) = 0$. E_1/M_0 is the energy per unit mass released by the thermonuclear explosion driving the Type Ia supernova, and $\delta(E_1/M_0)$ is the energy associated with the relatively slow radioactive decay of ^{56}Ni. The numerical coefficients in Eq. (6) take into account the build-up and subsequent decay of ^{56}Co. The decay times t_2 and t_3 are, of course, fixed by the nuclear physics and equal to $t_2 = 7.6 \times 10^5$ sec., $t_3 = 9.7 \times 10^6$ sec., respectively. By defining $E_1 = \nu E_g$ where $E_g = GM_0^2/\sqrt{2\pi}r_0$ is the gravitational potential energy, Eq. (7) can be rewritten as

$$yy'' + \frac{y'^2}{2} = \nu f(t) - (1 - \frac{1}{y}) \tag{8}$$

In Eq. (8), the primes are derivatives with respect to dimensionless time, t/τ with,

$$\tau = \left[3\sqrt{\pi/2}r_0^3/GM_0\right]^{1/2} \quad \text{or} \quad \tau = \left[3/\pi\sqrt{2}G\rho_0\right]^{1/2} \tag{9}$$

Eq. (8), along with Eq. (9), is the basic similarity model differential equation for the evolution of the density scale-height.

The equation governing the central temperature variation is given by Eq. (4). Converting $\ddot{y}$ to a derivative with respect to dimensionless time, y'', we find the central temperature θ to be

$$\theta^4 = \theta_0^4\left(yy'' + 4/\sqrt{2}y\right)/y^3 \qquad \theta_0^4 = \rho_0^2 r_0^2 \pi G/\sqrt{2}a \tag{10}$$

where, as above, the derivatives are with respect to dimensionless time.

Therefore, choosing a set of parameters and integrating Eq. (8), we can then use Eq. (9) to find the temperature as a function of (dimensionless) time. To get the energy radiated, we need the surface temperature. The SN expansion is very rapid, so that thermal diffusion is relatively unimportant in the early stages of the expansion. We can thus approximate the surface temperature by $\theta(t)\exp(-\eta^2/4)$ with $\eta = r_{\mathrm{surface}}/r_{\mathrm{s}}$. In later stages of the expansion there are corrections to this temperature which we discuss in the next section.

Numerical integration of Eq. (8) with Eq. (9) and the definitions given above is the procedure for calculations in our simplified similarity solution (MRb). The integrations were carried out on a PC using *Mathematica* (1999) with runs taking only a few seconds. We convert our luminosity calculation, $4\pi r_0^2\eta^2 y^2\sigma\theta_{\mathrm{st}}^4$, to absolute stellar magnitude using $L(M_{\mathrm{V}}) = 3.07 \times 10^{35}\exp(-0.921\,M_{\mathrm{V}})$ and, for the SCP data, an assumed Hubble constant $h = 65$km/s/Mpc. Finally, we apply a bolometric correction (BC) (actually a filter correction) similar to that used by the observers. The BC (V-band) we use is that given by Allen (1963),

$$\mathrm{BC} = -1.855 + 4.343\log_e\theta_{\mathrm{st}} + 2.5/\theta_{\mathrm{st}} \tag{11}$$

with the surface temperature θ_{st} measured in eV. This correction, which is positive, is added to the calculated magnitude M_{V}. The shape of the light curve is, of course, partially determined by the BC.

In Figure 1 we plot the light-curves of several Type Ia SNe produced by our model along with two sets of SCP data points from their website (SCP, 1998) (we refer to the upper curves as the 'orange' data set and the lower as the 'green' data set as they were color coded on the SCP website). We find that it is possible to fit our model to the SCP data points with two progenitors of the same mass and radius but having produced differing amounts of ^{56}Ni.

Now, is this the only fit to the data, or can we obtain satisfactory models using other parameters? We can formally fit the 'orange' data set equally well, with larger mass progenitors having the same central density, but as we show in Section 4 these scenarios can be ruled out because values of some of the parameters are not realistic. (Our earlier paper MRa, gives a complete and more detailed derivation of the similarity model; however, the attempt in MRa to compare results with the light-curve data was inaccurate because we did not include a bolometric correction.)

3. The Characteristic Time and the Similarity Model

Returning now to the characteristic time, τ in Eq. (9). Even though the earliest observed light from the SN is from a radius expanded some 10^6 fold, the time scale of the light curve is governed by a feature from its thermonuclear origin. The characteristic time is a measure of the time required for the star's mass to escape

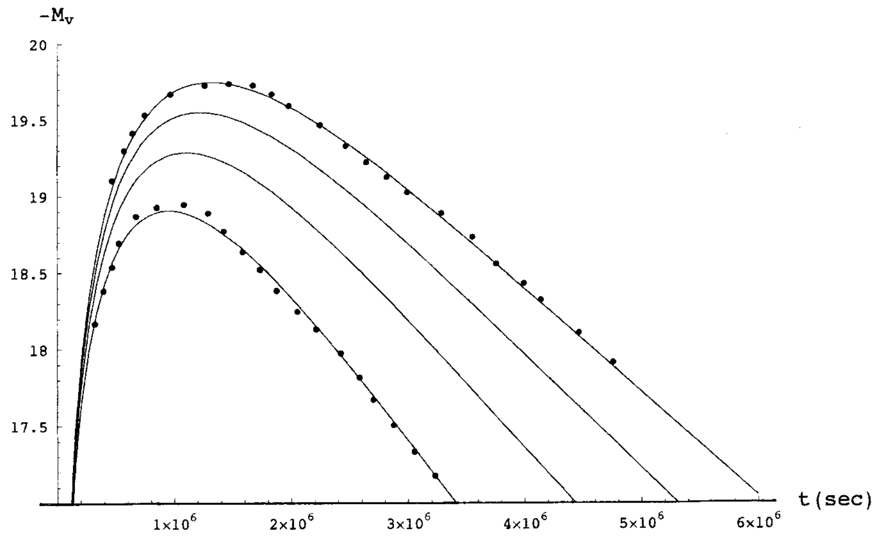

Figure 1. The SCP magnitude data plotted along with the MRb model fits for the 'orange' data (upper curve) and the 'green' data (lower curve). All of the model curves have $m = 1.4$, $\rho_0 = 3.7 \times 10^9 \mathrm{g/cm}^3$, $\nu = 1.52$, $t_1 = 100$ sec., and $\eta = 2$. The 'green' data set has ^{56}Ni fraction $\epsilon = 0.30$, the 'orange data set has the ^{56}Ni fraction $\epsilon = 0.83$. The intermidiate curves have $\epsilon = 0.49$ and $\epsilon = 0.62$. The relationship between ϵ and δ is discussed in the text.

its own gravitational potential well moving at the escape velocity. We prefer to call τ the 'dynamic time'. Interestingly, the dynamic time has an analog (with the same density scaling) in the cosmological expansion of the universe (Peacock, 1999). A typical value of τ is on the order of a millisecond; a rough measure of the escape velocity is r_0/τ.

Equation (9) shows that the dynamic time depends only upon the central density of the progenitor star. Since Eq. (8) is written in terms of the dimensionless time, we might expect light-curves for stars with ***the same central density*** but different masses and radii will produce similar light-curves. These light-curves would appear, in observation, to have roughly the same temporal shape but different amplitudes. This is what we would observe if all other parameters remain unchanged. However, changes in δ (or ϵ) also affect the 'width' and amplitude of the light-curve. This combination of effects is the reason behind the fact that so many light-curve observations along with an amplitude and time re-scaling, can be made to have the same shape.

The amplitude of the light-curve, in our model, depends primarily on the parameters ρ_0, ν, and δ; the parameter t_1 also affects the amplitude since a slower release puts more of the energy into internal energy, but this is a small effect. t_2, the radioactive ^{56}Ni decay time which is the main source of energy generation during the early stage of the expansion near the peak in luminosity, is believed to be the same for all Type Ia SNe. After about 20 to 30 days (from initiation) the radioactive daughter ^{56}Co begins to add significantly to the energy production. At later times there are other energy losses from the expanding star, e.g., from the well-known gamma-ray transparency effect (Arnett, 1996), so that modeling of the late phase expansion was not pursued.

4. Discussion

It is interesting that the dynamic time, which one expects to be associated with the early explosive expansion of the SN out of the it's own gravitational well, plays a dominant role in the system's subsequent evolution. We have made numerous integrations of our model equations and examined how the light-curve shapes depend upon various parameters. The fact that so many of the experimentally observed SN light-curves can be fit on one plot with only a small time-stretching factor is suggestive of some sort of threshold density effect. Our progenitor's central density ρ_0 turns out to be $3.7 \times 10^9 \mathrm{g/cm}^3$. Hoyle and Fowler (1960) and Arnett (1969) have postulated the occurrence of degenerative ignition in accreting white dwarfs at densities just over 10^9 g/cm^3. Our implied central density appears to support this idea.

Our model, which conserves mass, momentum, and energy, puts severe constraints on the shape of the early part of the light curve. For example, the combination of magnitude, timing of the peak, and overall shape of the light-curve can only be achieved with a restricted range of parameters. The parameter δ also affects the choice of these parameters since δ is determined by the total amount of ^{56}Ni generated. In fact, $\delta = 11.2r\epsilon/m\nu$ where m and r are the mass and radius (in solar units), and ϵ is the fraction of ^{56}Ni; with its radioactive energy available for heating and expansion. It is not easy to find different groupings of parameters that produce a specific observed light-curve. As we mentioned earlier, it is possible to fit formally the 'orange' data set to larger mass progenitors of the same central density. However, these models require roughly the same values of ν and δ, and this translates into ϵ values (^{56}Ni fraction) greater than one.

In Figure 1, we fit both the 'orange' and 'green' data sets using progenitors of the same mass and density ($m = 1.4$, $\rho = 3.7 \times 10^9$ g/cm^3) but with two different values of ϵ: 0.28 and 0.79. This suggests the following picture: most Type Ia SNe are different 'burn-up' fractions of a **unique mass** progenitor. This picture also supports a widely held view of Type Ia SNe, that they all have the same mass, that they are formed from accreting white dwarfs with mass just above the Chandrasekhar mass. Interestingly, if this scenario is indeed correct, then one can estimate the burn-up fraction from the 'width' of the light-curve. We should mention that the salient feature of this scenario, namely, that the SN with the smaller amount of ^{56}Ni shows a smaller half-width in its light-curve, agrees with the useful (but 'empirical') observation of Phillips (1993) as well as a parametric study by Pinto and Eastman (2000).

Can we learn anything more from our model? The gravitational energy per unit mass of our progenitor is $E_g/M_0 = 6.5 \times 10^{17}$ ergs/g. Most of the energy νE_g in the thermonuclear reaction must come from either the nuclear physics or gravitational energy. We find the right order of magnitude from nuclear energy obtained in the conversion of carbon and oxygen to nickel, but the νE_g value is a little high and suggests that a small fraction of the star's mass may be left behind as a neutron star.

In conclusion, our results indicate that the homogeneity of observed Type Ia light-curves results from (1) a threshold central density for the progenitor which controls the 'dynamic' time scale, and (2) a limited size/mass parameter domain for the progenitor. Furthermore, our similarity model shows that mass, momentum, and energy conservation severely limits the domain of SN progenitor parameters able to reproduce the observed Type Ia light-curves, so certain ranges of progenitor parameters must be excluded on this basis.

References

Allen, C.W.: 1963, *Astrophysical Quantities*, The Athlone Press, London, p. 191.

Arnett, W.D.: 1996, *Supernovae and Nucleosynthesis*, Princeton University Press, pp. 451–458.

Arnett, W.D.: 1969, A possible model of SNe: Detonation of ^{12}C, *Astrophys. & Space Sci.* **5**, 180–212.

Goldhaber, G. et al.: 2001, Timescale stretch parameterization of Type Ia supernova B-band light curves, accepted for publication in *AJ*, (also references therein).

Hoyle, F. and Fowler, W.A.: 1960, Nucleosynthesis in Supernovae, *AJ* **132**, 565.

Mayer, F.J. and Reitz, J.R.: 2001, A similarity model for supernovae light curve calculations, *Astr. & Astrophys.* **366**, 174–177.

Mathematica-V4: 1999, Wolfram Research, Inc., Champaign, IL 61820.

Peacock, J.A.: 1999, *Cosmological Physics*, Cambridge University Press, p. 84.

Pinto, P.A. and Eastman, R.G.: 2000, The Physics of Type Ia Supernova Light Curves. I. Anaytic Results and Time Dependence, *AJ* **530**, 744–756.

Phillips, M.M.: 1993, *AJ* **413**, L105–L108.

Reiss, A.G. et al., 1998, Observational evidence from supernovae for an accelerating universe and a cosmological constant, *AJ* **116**, 1009–1038 (also references therein).

The Supernova Cosmology Project: 1998, Lawrence Berkeley Laboratory. Data from the SCP poster from the January meeting of the Amer. Astron. Soc. on the website: http://www-supernova.lbl.gov